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AN UNSUCCESSFUL ATTEMPT TO REDUCE THE PHASE OSCILLATION FREQUENCY BY A PHASE SHIFT BETWEEN ACCELERATING CAVITIES

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The purpose of this note is to investigate the effect on the longitudinal motion of a particle that is accelerated by rf cavities that have a relative phase difference. Consider a particle that arrives at rf cavities No. 1 and 2 when the phase of the cavities are ϕ_1 and ϕ_2. The energy gain per turn is given by

$$\Delta E/\Delta n = V/2 \sin \phi_2$$

where V is the sum of the peak voltage of each cavity. If we define the average phase of the cavities by ϕ and the phase difference by 2δ, i.e., $\phi = 1/2(\phi_1 + \phi_2)$ and $\phi_1 - \phi_2 = 2\delta$, then we have

$$\Delta E/\Delta n = V \cos \delta \sin \phi$$

For the case where we replace the difference equations by differential equations, we obtain

$$\frac{dW}{dt} = V \cos \delta \,(\sin \phi - \sin \phi_s),$$

$$\frac{d\phi}{dt} = \frac{2\pi h f^2}{\gamma E_0} K W,$$

where h is the harmonic number, ϕ_s is the average phase of a synchronous particle, f is the revolution frequency, $K = (f/E)^{-1}\, df/dE$, and W is the canonical momentum $\frac{E - E_s}{f}$

These equations of motion may be derived from the following Hamiltonian:

$$H = - \frac{2\pi h f^2 K}{\gamma E_0} \frac{W^2}{2} + \overline{V} \;\; \cos\phi - \cos\phi_s + (\phi - \phi_s) \sin\phi_s$$

with $\overline{V} = V \cos\delta$.

The only result of having two rf cavities with a relative phase difference of $2\,\delta$ is to decrease the total effective voltage both for accelerating the synchronous particle and for focusing the nonsynchronous particle. Thus, this is not an effective method of reducing the phase oscillation frequency since it also reduces the bucket area.